ERRATUM TO: ON CHARACTERISTIC CLASSES OF EXOTIC MANIFOLD BUNDLES

MANUEL KRANNICH

Abstract. We correct the proof of Corollary C. All results of the paper are unaffected.

In the proof of Corollary C of [Kra19b], we misquoted a result of Sullivan which lead to the erroneous conclusion that the mapping class group $\pi_0 \operatorname{Diff}^+(W_g)$ of the manifold $W_g = \sharp^g S^n \times S^n$ is residually finite for $2n \geq 6$. This is indeed often not the case, as shown in [KRW19]. However, as we shall explain below, the ideas of [KRW19] can be combined with results from [Kre79] and [Kra19a] to correct the proof of Corollary C.

Proof of Corollary C. From the part of the original argument that is not affected by our misquotation, together with the proof of Proposition 3.12 of the same paper, we see that it suffices to show that the mapping class group $\pi_0 \operatorname{Diff}(W_g, D^{8k+2})$ is for $k \geq 0$ not (abstractly) isomorphic to the cokernel of the map

$$\mathbf{Z}/2 \xrightarrow{t_*} \pi_0 \operatorname{Diff}(W_g \sharp \Sigma_\mu, D^{8k+2}) \cong \pi_0 \operatorname{Diff}(W_g, D^{8k+2}), \tag{1}$$

where t_* is as in the proof of Corollary C and the isomorphism is induced by conjugation with a diffeomorphism

$$W_g \sharp \Sigma_\mu \backslash \operatorname{int}(D^{8k+2}) \cong W_g \backslash \operatorname{int}(D^{8k+2}) \tag{2}$$

as explained in Lemma 4.1, which we can choose to be the identity on an embedded disc $\widetilde{D}^{8k+2}$ in both manifolds, disjoint from the already chosen one. The homotopy sphere $\Sigma_\mu \in \Theta_{8k+2}$ is the unique one corresponding to Adams' element $\mu_{8k+2} \in \pi_{8k+2}\mathbf{S}$ (see the proof of Prop. 3.12). Extending diffeomorphism by the identity gives a map

$$\Theta_{8k+3} \cong \pi_0 \operatorname{Diff}_\partial(\widetilde{D}^{8k+2}) \longrightarrow \pi_0 \operatorname{Diff}(W_g \sharp \Sigma_\mu, D^{8k+2})$$

and an analogous one for W_g instead of $W_g \sharp \Sigma_\mu$. By our choice of (2), these two morphisms commute with the isomorphism in (1) and are injective by [Kre79, Prop. 3]. From the discussion in [Kre79, p. 657], we furthermore conclude that the image of t_* is generated by a certain homotopy sphere $\Sigma_{W_g \sharp \Sigma_\mu} \in \Theta_{8k+3}$ which is nontrivial by [Kre79, Thm 3 c)] since $\eta \cdot \mu_{8k+2} \in \pi_{8k+3}\mathbf{S}$ is so (compare the proof of Prop. 3.12). But $\eta \cdot \mu_{8k+2}$ is contained in $\operatorname{im}(J)_{8k+3}$, so the image of $\Sigma_{W_g \sharp \Sigma_\mu}$ in $\operatorname{coker}(J)_{8k+3}$ vanishes and hence $\Sigma_{W_g \sharp \Sigma_\mu}$ must be a nontrivial element in $\mathrm{bP}_{8k+4} \subset \Theta_{8k+3}$. Consequently, it suffices to show that the group $\Gamma_{g,1} \coloneqq \pi_0 \operatorname{Diff}(W_g, D^{8k+2})$ cannot be isomorphic to its quotient $\Gamma_{g,1}/\langle \Sigma \rangle$ for any nontrivial element $\Sigma \in \mathrm{bP}_{8k+4}$. The case $g = 0$ is clear since $\Gamma_0 = \Theta_{8k+3}$. In the case $g = 1$, by a combination of Theorems 2.2 and 3.22 [Kra19a], there is an isomorphism $\Gamma_{1,1} \cong \Theta_{8k+3} \times (A \rtimes G)$ for a finitely generated abelian group A with an action of a finite index subgroup $G \subset \mathrm{SL}_2(\mathbf{Z})$. In particular, the group $\Gamma_{1,1}$ is Hopfian by the argument in the original proof of Corollary C, so the case $g = 1$ is fine. To settle the case $g \geq 2$, we use the *finite residual* $\operatorname{fr}(G)$ of a group G, which is the intersection of all its finite index subgroups. Since $\operatorname{fr}(-)$ is functorial, it suffices to show that $\operatorname{fr}(\Gamma_{g,1})$ and $\operatorname{fr}(\Gamma_{g,1}/\langle \Sigma \rangle)$ are not isomorphic. We claim that $\operatorname{fr}(\Gamma_{g,1}) = \mathrm{bP}_{8k+4}$ for $g \geq 2$, which would imply the claim because of the following elementary lemma.

Lemma. *Let G be a group and $H \subset G$ a normal subgroup. If $H \subset \operatorname{fr}(G)$, then the canonical map $\operatorname{fr}(G) \to \operatorname{fr}(G/H)$ induces an isomorphism $\operatorname{fr}(G)/H \cong \operatorname{fr}(G/H)$.*

It was already shown in [KRW19] that $\mathrm{fr}(\Gamma_{g,1})$ is isomorphic to bP_{8k+4} for k odd and $g \geq 5$, using [GRW16]. To enhance this result to an equality and general k and $g \geq 2$, we use the same strategy as in [KRW19], but replace the use of [GRW16] by Theorems 2.2 and 3.22 of [Kra19a] which imply that there is a pullback of extensions

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \Theta_{8k+3} & \longrightarrow & \Gamma_{g,1} & \longrightarrow & (S\pi_n\,\mathrm{SO}(n) \otimes \mathbf{Z}^{2g}) \rtimes \mathrm{Sp}_{2g}^q(\mathbf{Z}) & \longrightarrow & 0 \\ & & \| & & \downarrow & & \downarrow{\scriptstyle \mathrm{pr}_2} & & \\ 0 & \longrightarrow & \Theta_{8k+3} & \longrightarrow & G & \longrightarrow & \mathrm{Sp}_{2g}^q(\mathbf{Z}) & \longrightarrow & 0 \end{array} \tag{3}$$

with G being defined by the pushout of extensions

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \mathbf{Z} & \longrightarrow & E_g & \longrightarrow & \mathrm{Sp}_{2g}^q(\mathbf{Z}) & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle \Sigma_P} & & \downarrow & & \| & & \\ 0 & \longrightarrow & \Theta_{8k+3} & \longrightarrow & G & \longrightarrow & \mathrm{Sp}_{2g}^q(\mathbf{Z}) & \longrightarrow & 0, \end{array} \tag{4}$$

where the extension E_g is classified by the class $\frac{\mathrm{sgn}}{8} \in \mathrm{H}^2(\mathrm{Sp}_{2g}^q(\mathbf{Z}); \mathbf{Z})$ of [Kra19a, Def. 3.16] and $\Sigma_P \in \Theta_{8k+3}$ is the Milnor sphere—the usual generator of bP_{8k+4}. In [KRW19], it is explained that $\mathrm{fr}(E_g) = \mathbf{Z}$ for $g \geq 2$, which implies that $\mathrm{fr}(E) = \langle \Sigma_P \rangle = \mathrm{bP}_{8k+4}$, using (4) and $\mathrm{fr}(\mathrm{Sp}_{2g}^q(\mathbf{Z})) = 0$. Combined with (3) and $\mathrm{fr}(S\pi_n\,\mathrm{SO}(n) \otimes \mathbf{Z}^{2g}) \rtimes \mathrm{Sp}_{2g}^q(\mathbf{Z})) = 0$, one checks that $\mathrm{fr}(\Gamma_{g,1}) = \mathrm{bP}_{8k+4}$, which finishes the proof.

References

[GRW16] S. Galatius and O. Randal-Williams, *Abelian quotients of mapping class groups of highly connected manifolds*, Math. Ann. **365** (2016), no. 1-2, 857–879.

[Kra19a] M. Krannich, *Mapping class groups of highly connected* $(4k+2)$*-manifolds*, arXiv:1902.10097.

[Kra19b] ———, *On characteristic classes of exotic manifold bundles*, to appear in Math. Ann. (2019).

[Kre79] M. Kreck, *Isotopy classes of diffeomorphisms of* $(k-1)$*-connected almost-parallelizable* $2k$*-manifolds*, Algebraic topology, Aarhus 1978, Lecture Notes in Math., vol. 763, Springer, Berlin, 1979, pp. 643–663.

[KRW19] M. Krannich and O. Randal-Williams, *Mapping class groups of simply connected high-dimensional manifolds need not be arithmetic*, available through the authors' websites.

Email address: krannich@dpmms.cam.ac.uk

Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WB, UK